Creating Support Structures to Help Teachers Engage in Formative Assessments

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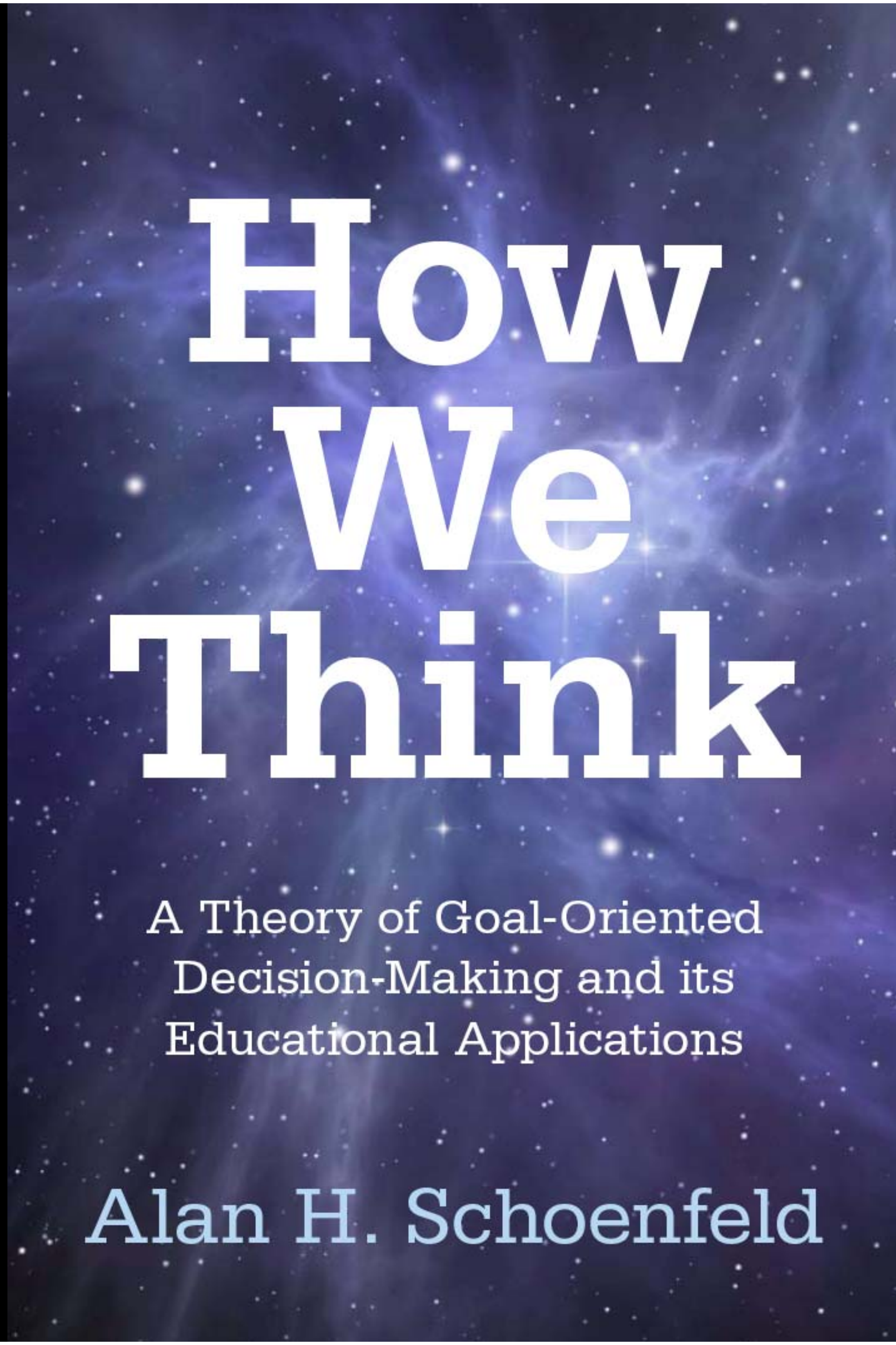
First, the backdrop:


How We Think
A Theory of Goal-Oriented Decision-Making and its Educational Applications
Alan H. Schoenfeld

The key assertion in the book:

Teachers' in-the-moment decision-making is a function of their knowledge/resources, goals, and beliefs/orientations.

Theoretical claim: If I know “enough” about a teacher’s resources, goals, and orientations, I can model his/her actions and explain them on a line-by-line basis.

Practical Note:

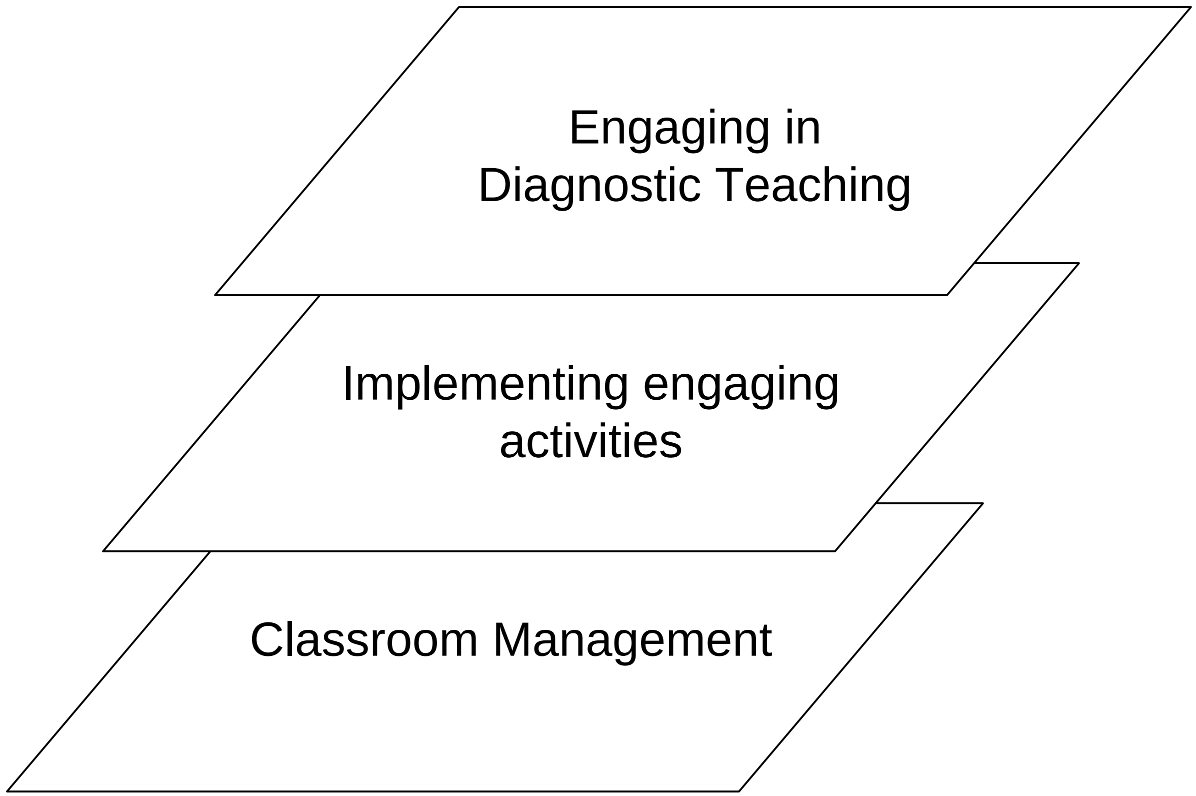
The three driving factors are critically important - they're the "make or break" elements of effective teaching.

For example, consider what a teacher might do when a student writes…

$$(a+b)^2 = a^2 + b^2$$

A hypothesized development space: Dimensions of teacher growth

Three Planes of Teaching Activity



A Typical Beginning Teacher Profile

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

A Typical Accomplished Teacher Profile

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

A Typical Expert Teacher Profile

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

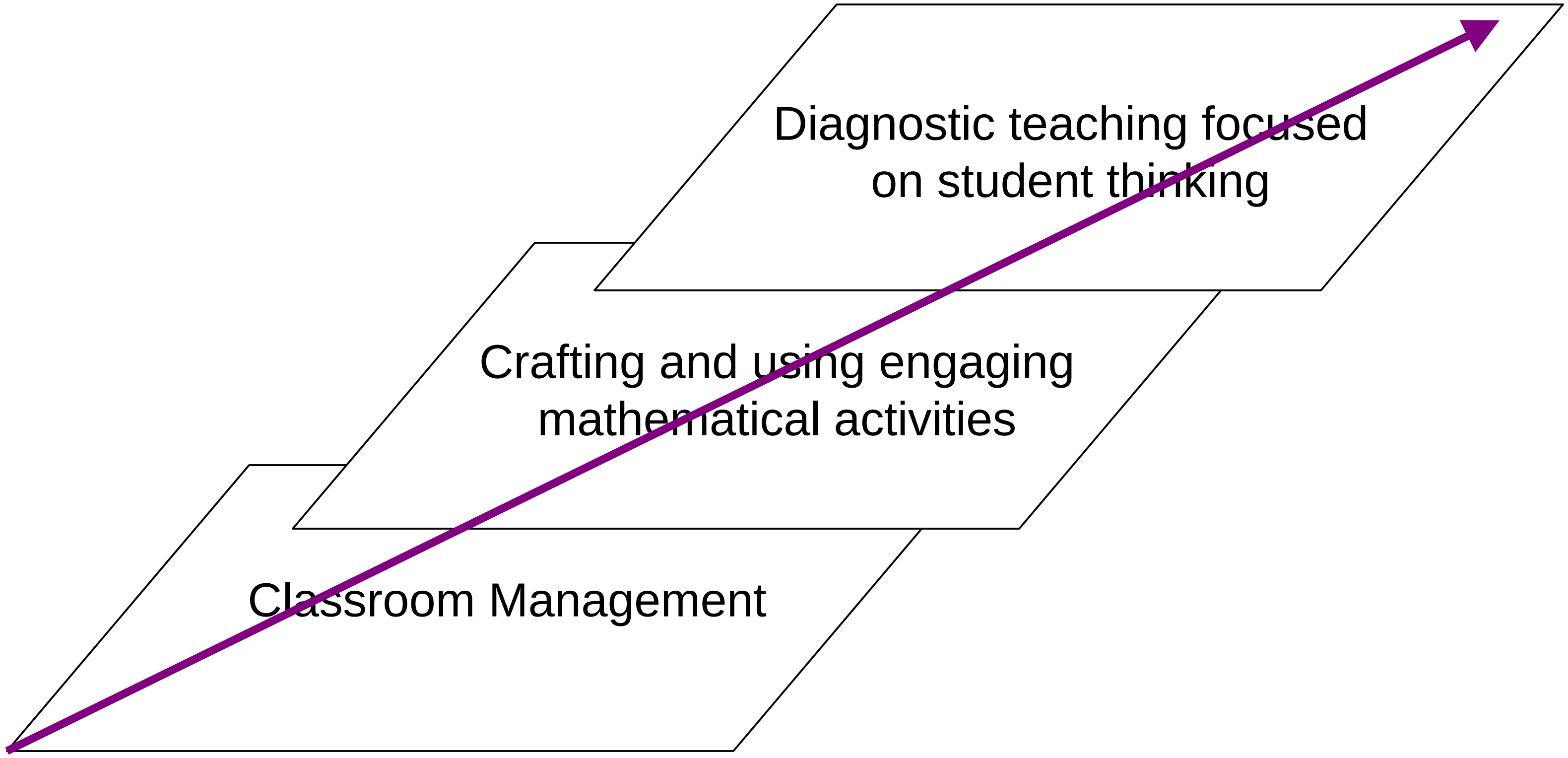
Our goal for PD:

Accelerate teachers' movement along this pathway.

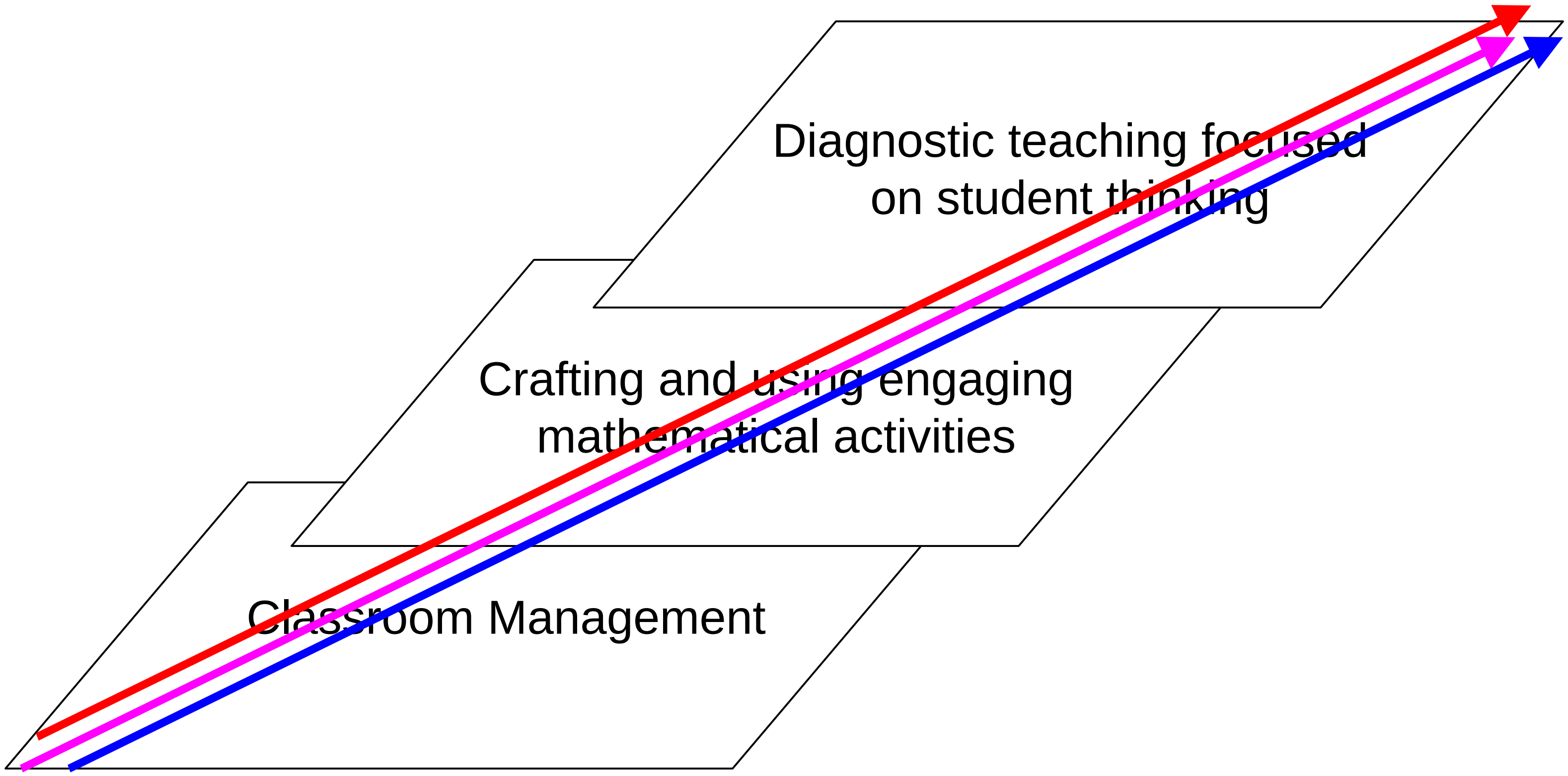
It’s a long slow road, if you take the bottom lines from the research seriously.

Imagine a single teacher’s metaphorical trajectory through this space:

A particular teacher's metaphorical trajectory:

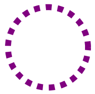


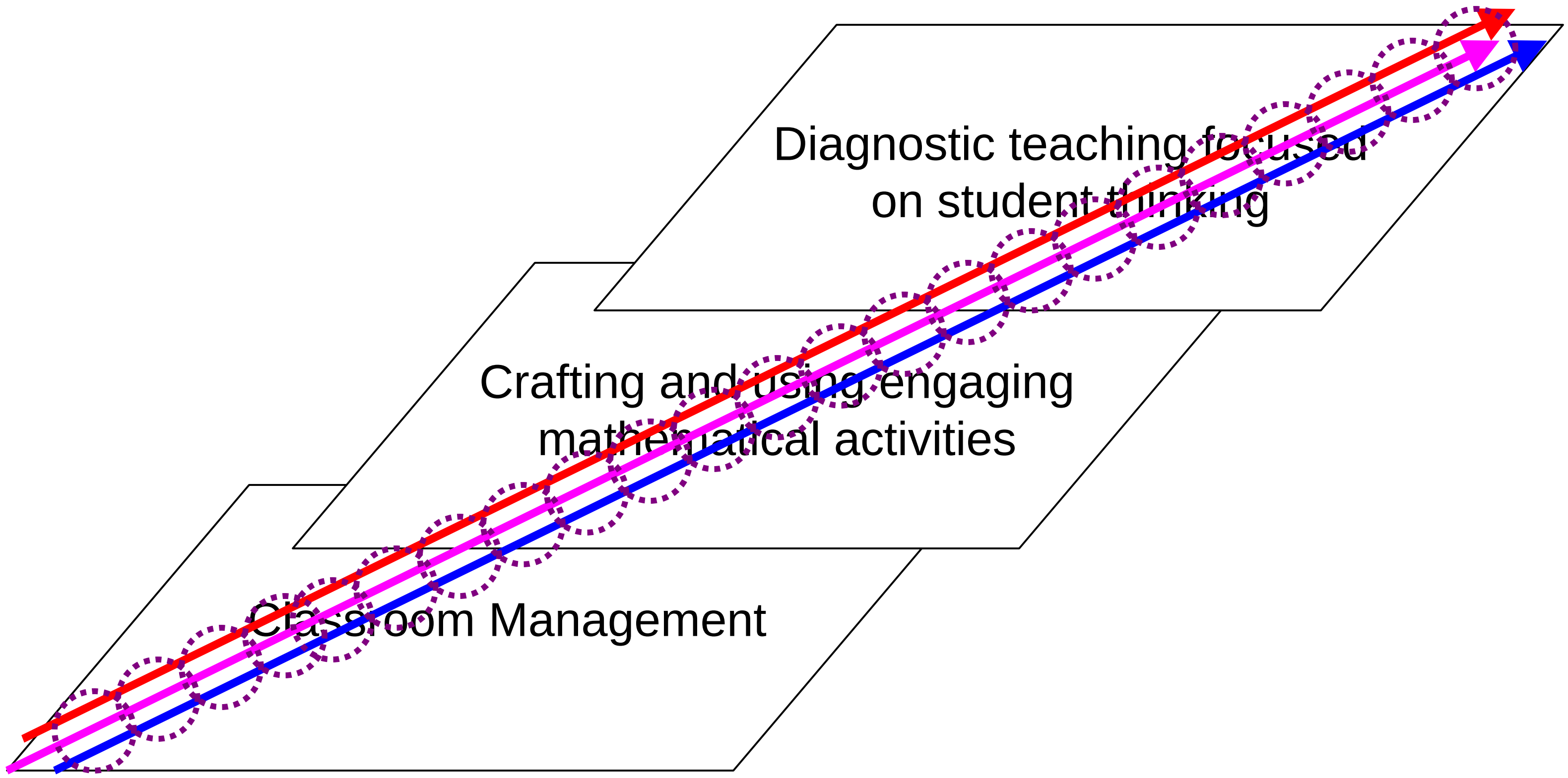
As he or she grows, that teacher develops an evolving set of knowledge & resources, goals, and beliefs & orientations:



But … the point of “How we Think” is that knowledge and resources, goals, and beliefs and orientations are all deeply intertwined.

For example, if the teacher decides to teach in a way that is responsive to student thinking, he or she may not have the knowledge and resources to do so! Hence…

Knowledge & resources, goals, and beliefs & orientations are clustered  and evolve slowly.



In sum: the evolution of professional competence is necessarily slow.

Development of expertise in any field takes 5,000-10,000 hours of focused, reflective action!

Practical issues

The central issue of professional development is how to help teachers, over time, develop some of the skills and understandings we value.

My focus here is on formative assessment or “diagnostic teaching.”

What is formative assessment?

Summative assessments show what students "know and can do" *after* instruction. That's important, but it's too late to help the students learn.

Formative Assessments reveal students' current understandings so you can help them improve.

Important Background Issues

1. Formative assessment is *not* summative assessment given frequently!
2. Scoring formative assessments rather than or in addition to giving feedback destroys their utility (Black & Wiliam, 1998: “inside the black box”)
3. This is HARD for teachers to do, so we also need to provide teachers with tools to help them make effective use of the information that the assessments provide.

How can you support teachers in doing formative assessments?

Here’s one way:

The formative assessment lesson:

A rich “diagnostic” problem

and

Things to do when you see the results of the diagnosis.

Here are two examples of challenges, tasks to reveal issues, and treatments.

Challenge:

We know that students have many graphing misconceptions, e.g., confusing a picture of a story with a graph of the story in a distance-time graph.

Before the lesson, we point to typical student misconceptions and offer suggestions about how to address them.

Common issues	Suggested questions and prompts
Graph interpreted as a picture E.g. The student assumes that as the graph goes up and down, that Tom's path is going up and down. E.g. The student assumes that a straight line on a graph means that the motion is along a straight path. E.g. The student thinks the negative gradient means Tom has taken a detour.	• *If a person walked in a circle around their home, what would the graph look like?* • *If a person walked at a steady speed up and down a hill, directly away from home, what would the graph look like?* • *In each section of his journey, is Tom's speed steady or is it changing? How do you know?* • *How can you work out Tom's speed in each section of the journey?*
Graph interpreted as speed v time The student has interpreted a positive gradient as speeding up and a negative gradient as slowing down.	• *If a person walked for a mile at a steady speed, away from home, then turned round and walked back home at the same steady speed, what would the graph look like?* • *How does the distance change during the second section of Tom's journey? What does this mean?* • *How does the distance change during the last section of Tom's journey? What does this mean?* • *How can you tell if Tom is travelling away from or towards home?*

Then we scaffold the lesson structure…

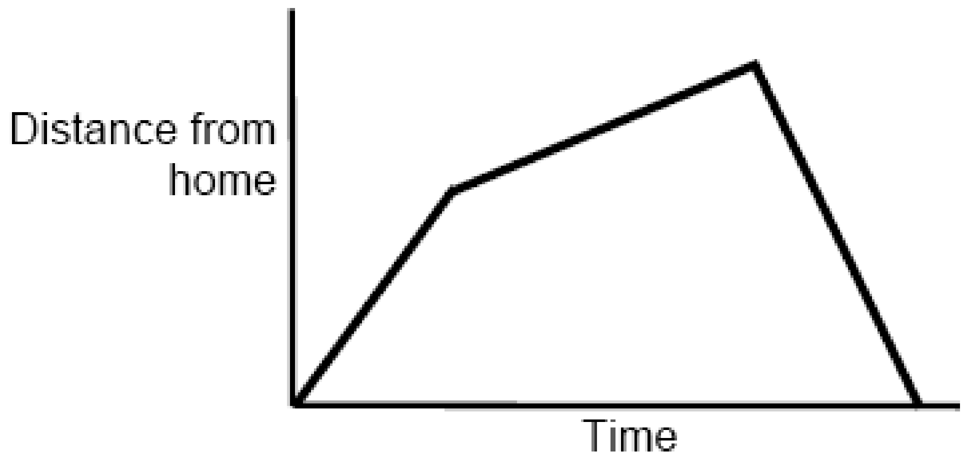
e.g., the diagnostic part…

Whole class introduction: Interpreting and sketching graphs (10 minutes)

Throughout this activity, encourage students to articulate their reasoning, justify their choices mathematically and question the choices put forward by others.

This introduction will provide students with a model of how they should work with their partners throughout the first small group activity.

Use the projector resource *Matching a graph to a story* to show the class:



A.
Tom took his dog for a walk to the park. He set off slowly and then increased his pace. At the park Tom turned around and walked slowly back home.

B.
Tom cycled east from his home up a steep hill. After a while the slope eased off. At the top he raced down the other side.

C.
Tom went for a jog. At the end of his road he bumped into a friend and his pace slowed. When Tom left his friend he walked quickly back home.

Ask students to match the correct story to the graph. They are to write down at least two reasons to support their decision.

After two or three minutes ask students who selected Option A to raise their hands. Ask one or two to justify their choice. You may wish to use some of the questions on the sheet *Suggested questions and prompts* on the page 10 to encourage students to justify their choices and others to challenge their reasoning.

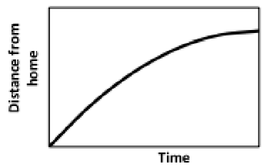
Repeat this with Options B. and C. Even if explanations are incorrect or only partially correct write them next to the appropriate section of the graph. Encourage students to challenge these interpretations and then replace them with new ones.

And then we provide a full-blown task that makes use of what the teacher has learned about student understandings, and helps to address them:

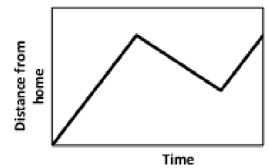
Instructional Task: Card Sort

Card Set A: Distance-Time Graphs

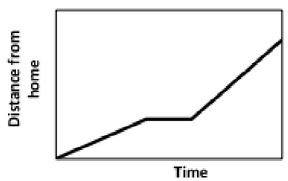
A.



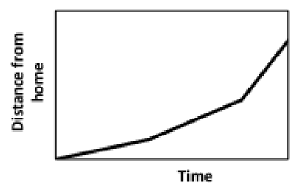
B.



C.



D.



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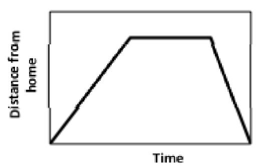
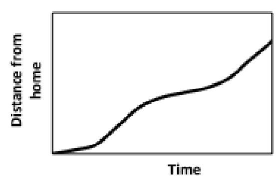
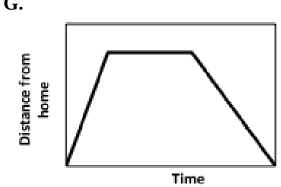


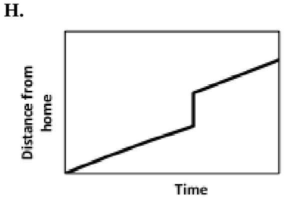
F.



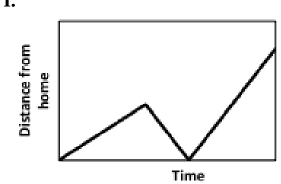
G.



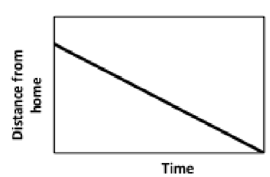
H.



I.



J



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Card Set B: Interpretations

1. Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.	2. Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top and then ran quickly down the other side.
3. Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.	4. Tom walked slowly along the road, stopped to look at his watch, realized he was late, then started running.
5 Tom left his home for a run, but he was unfit and gradually came to a stop!	6. Tom walked to the store at the end of his street, bought a newspaper, then ran all the way back.
7. Tom went out for a walk with some friends when he suddenly realised he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.	8. This graph is just plain wrong. How can Tom be in two places at once?
9. After the party, Tom walked slowly all the way home.	10. Make up your own story!

Card Set C: Tables of data

A.

Time	Distance
0	0
1	40
2	40
3	40
4	20
5	0

B.

Time	Distance
0	0
1	10
2	20
3	40
4	60
5	120

C.

Time	Distance
0	0
1	18
2	35
3	50
3	85
5	120

D.

Time	Distance
0	0
1	40
2	80
3	60
4	40
5	80

E.

Time	Distance
0	0
1	20
2	40
3	40
4	40
5	0

F.

Time	Distance
0	0
1	30
2	60
3	0
4	60
5	120

G.

Time	Distance
0	0
1	20
2	40
3	40
4	80
5	120

H.

Time	Distance
0	0
1	45
2	80
3	105
4	120
5	125

I

Time	Distance
0	120
1	96
2	72
3	48
4	24
5	0

J. Make this one up!

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

K.

Time	Distance
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Accompanied/followed by…

A set of activities tuned to specific student (mis)understandings, e.g.,

“generate data corresponding to [a particular story] and then graph them.”

Another Challenge:

Kids “learn” the point-slope formula for a line, and sometimes other formulas, but they typically can’t apply them and are often stymied if the information they’re given isn’t in either point-slope or two-point form (if they’re to graph it.) Here’s a task:

WHO STOLE THE APPLE PIE?
The Mystery…

Ms. Lee lives 24 miles north of Oakville on the way to Albany. She baked an apple pie and set it out on her windowsill at 3:00 pm to cool. She went to get it at 6:30 pm and found it had been STOLEN!

Tom, Dora, Harry, and Joan, who all live on the road between Oakville and Albany, are the main suspects. Ms. Lee needs you to use the following information to solve the mystery of who stole the pie, and find the thief.

THE BUS DRIVER: A **bus** left Oakville at 4:00 p.m. going nonstop to Albany, 30 miles north. The bus arrived in Albany at 4:45 p.m., waited 15 minutes, and then returned to Oakville at the same rate. The driver saw the apple pie on Ms. Lee's windowsill as she drove north, but it was gone when she passed by on the way south back to Oakville.

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And here are some of the support materials.

WHO STOLE THE APPLE PIE?

Note: Materials for students, including the task and question sheets, are at the end of this packet.

What this lesson is about

This lesson provides an opportunity to see how well students can graph and interpret a complex story problem involving linear functions. It focuses on things that we know caus students difficulty, and sets the stage for addressing those difficulties. It involves a brief s up the day before the lesson (Ņthe mysteryÓ) and two class periods. It includes four sets o questions that probe student understanding and give them an opportunity to demonstrate what they know. It is best placed about 2/3 of the way through SFUSD Unit 2 (Linear Functions, Multiple Representations, and Problem Solving).

The Core Mathematics

Students often have difficulty applying graphs to various contexts. Here we expect them to interpret a graphical system with non-standard axes, which can be a challenge. Similarly, students can typically graph a line given two points or point and slope Š but minor variations throw them. This task asks students to understand and graph stories about motion at constant speed, and it asks them to generate line segments given *any* two pieces of information (e.g., a starting point and a rate of speed for given time intervals. This lesson asks students to:

1. use Ņclock timeÓfor the x-axis, so that a point on the graph indicates that someone or something is at a specific place at a specific time;
2. understand that in this context, the slope of a line segment indicates its speed;
3. interpret line segments (including horizontal line segments) appropriately;
4. accurately graph a line segment given *any* two pieces of information about it;
5. interpret the intersection of two lines as meaning simultaneity Š that the two people represented by the lines are at the same time at the same place; and
6. be able to explain all of the above.

These are central aspects of the Grade 8 Common Core Standards.

Expected student difficulties

1. Students are likely to have difficulty seeing that a fixed object, such the pie on Ms. LeeÕs windowsill or the bus station in Albany, is represented as a horizontal line segment on a distance-versus-time graph.
2. The x-axis represents time. Some students may not be able to locate Ņfractional timesÓ (e.g., 3:30) as points on the axis, and some may be confused because the x-values do not start at 0. Similarly, that some peopleÕs trajectories start in the ŅmiddleÓ of the graph may be confusing.
3. Understanding that the coordinate pair (time, location) corresponds to a where an object is a some point in time Š e.g., answering the question ŅWhere is everyone at 4:15 PM?
4. Interpreting rates such as 18 mph, especially when a person is traveling for less than 1 hour.
5. Understanding simultaneity Š that when two graphs cross, the two things represented are at the same place, at the same time.

- Seeing that steeper lines mean greater speed.

Materials and Class Setup:

Each student will need copies of the following handouts:

Who Stole the Apple Pie? The Mystery for the Launch.
Who Stole the Apple Pie? (with graphing instructions) for the lesson.
Who Stole the Pie Graph (There should be extra copies in case students make mistakes and want to try the graph again).
Getting Started Questions and Question Set I (can be front to back).
Question Sets II and III.

Each student needs access to:
A ruler or straight edge
Colored pencils: BLUE, GREEN, RED, ORANGE, PURPLE

This lesson assumes that students will be working together in groups of 2, 3, or 4 students.

Lesson Sequence

This lesson will typically start with a brief ŅlaunchÓ the day before students are fully engaged with it, and will then take two class periods.

The launch introduces students to the story. It motivates them and challenges them to solve it Š but does not tell them what mathematical tools they will be using. The power of graphing will be revealed during the lesson.

Goals for the first class period are to make sure that students understand the story (Getting Started Questions help with this), to introduce them to the graph, and to have them complete both the graph and Question Set I. Students may begin to answer the questions on Question Set II. You may want to assign all or part of these questions for homework, in preparation for class discussion the second day.

The goal on the second day is to have a discussion about how students drew the graphs, to solve the mystery, and to discuss the questions from Question Set II using the graph. Question Set III asks students to extend and apply their knowledge from this problem. The graph of EdwardÕs trip provides an opportunity to see how students use location and rate to describe the graph.

Important Note:

Question Sets I, II, and III form the basis for the Ņformative assessmentÓ part of the lesson. They are intended to reveal student understandings and misunderstandings, and to provide opportunities for students to explain what they understand. StudentsÕ answers to these questions, in small groups and in whole class discussion, provide the opportunity to see what they know and to build on it, working toward a collective solution to the problem.

Lesson Plan

Launch:

The day before the lesson, tell the students that they are going to use mathematics to help solve a mystery. Hand out **Who Stole the Apple Pie?** and tell the students that they are welcome to try to solve the mystery now! Say enough to entice the students into thinking about the mystery, but donÕt say how it will be solved. (See teacher-to-teacher notes below.)

Day 1:

Focal Issues and goals:

1. Learning the value of graphs as a representation, and how to interpret stories as graphs.
2. Learning to persevere in working complex problems.
3. Learning to generate the graph of a line segment, given *any two* pieces of information about it.
4. Interpreting features of graphs Š horizontal line segments, points of intersection of two lines.

- Learning to construct viable mathematical arguments and critique the reasoning of others.

Warm-Up (< 10 minutes)

Re-introduce the ***Who Stole the Apple Pie?*** story.

Hand out the ***Who Stole the Pie Graph*** and ***Getting Started Questions***, and have the class work through them.

Student investigation I (< 30 minutes)

1. Students work in small groups to produce the graphs representing the trajectories of Tom, Dora, Harry, and Joan.

2. The teacher circulates through the class, asking probing questions designed to check for understanding, scaffold their work, push for justifications. (See ***ŅTeacher Questions for Student Investigations,Ó***which are printed right before the handout materials.)

Student investigation II (to end of period)

3. When students have completed the graphs appropriately, have them work ***Question Set I.***

4. Those students who have finished ***Question Set I*** can begin working on ***Question Set II.***

- ***Question Set II*** is assigned for homework, as a basis for Day 2Õs activities.

Day 2:

Focal Issues and goals: Building on class discussions from Day 1, to focus on conceptual issues that need work.

Warm-Up (< 10 minutes): Have students write a response to the following question:

What different types of information did we use to draw the graphs in the Who Stole the Pie Problem? (*You may want to have the students hand in their responses. If students take this task seriously, the diagnostic information will be very useful for subsequent lessons.*)

Student investigation III (< 20 minutes)

Students work in pairs or teams/tables, discussing and comparing their answers to ***Question Set II***. Teacher circulates, identifying issues for final discussion. If things are moving smoothly, teacher may assign ***Question Set III***.

Concluding Discussion, Closure and homework.

Teacher orchestrates whole class discussion, displaying final graphs. Students are asked to explain who the thief is, and how they know.

If time permits, students work on ***Question Set III***; if not, it is assigned for homework.

As extension, the teacher can ask questions such as: ŅThe directions for this problem said to assume that everyone traveled at constant rates. Why is this important?Ó

Teacher-to-Teacher Notes

1. Without the use of graphing, this problem is VERY hard to solve! ThatÕs why we give it as a ÒteaserÓ the day before. If the students see how hard it is to solve without the graphs, theyÕll appreciate the use of the graphs when they work through the problem.

2. There are a lot of words in this problem. ThatÕs deliberate. Our goal is to help students with second language or other linguistic challenges to make sense of the problem situations, rather than simplifying the language into Òbite-sizeÓ pieces that no longer represent typical spoken or written English.

- The question sets are written so that they will reveal student misunderstandings. Extra questions below are intended to do the same. A series of pointed questions will often get students to see the correct mathematics; or when they explain what they have done correctly, it will be reinforced. Then they will remember it better.

We also offer these resources:

- Sample student work and issues to consider
- Follow-up activities, questions, and resources
- Issues related to disrict and national goals and standards
- Sample formative questions
- Student materials

Teacher Questions to check for understanding, scaffold, justify, etc.

If a student's work is correct: "Can you explain how you got XXX?"

Problem 1: Student cannot figure out the fractional times:

Ask the student(s) if they could decide what could be halfway between 3:00 and 4:00 and go from there.

Problem 2: Understanding that the points on the graph indicate a place at a specific time.

Ask the student to re-read the descriptions of the x and y axis and then ask them to point to where certain people are at certain times.

Problem 3: Understanding that people's trajectories might start in the middle of the graph.

Use the real world as an example, e.g., that both of us are in this room right now but I left for work at 7:15 and you didn't leave your house until 8:30

Problem 4: Understanding what a horizontal line segment means.

Quickly sketch axes representing "time" and "distance from school." Ask the student to plot how far from home he/she was at the beginning of the period, 5 minutes later, 10 minutes later, etc.

Problem 5: Interpreting MPH. If the person is not traveling for the full hour.

Ask: If you leave here driving at 60 MPH, how far have you gone in $\frac{1}{2}$hour? 15 minutes? 45minutes? Etc.

Problem 6: Understanding Simultaneity

(If it occurs): Ask what the point of intersection means. What does the point mean seen as a point on Line 1?, then the point on line 2.

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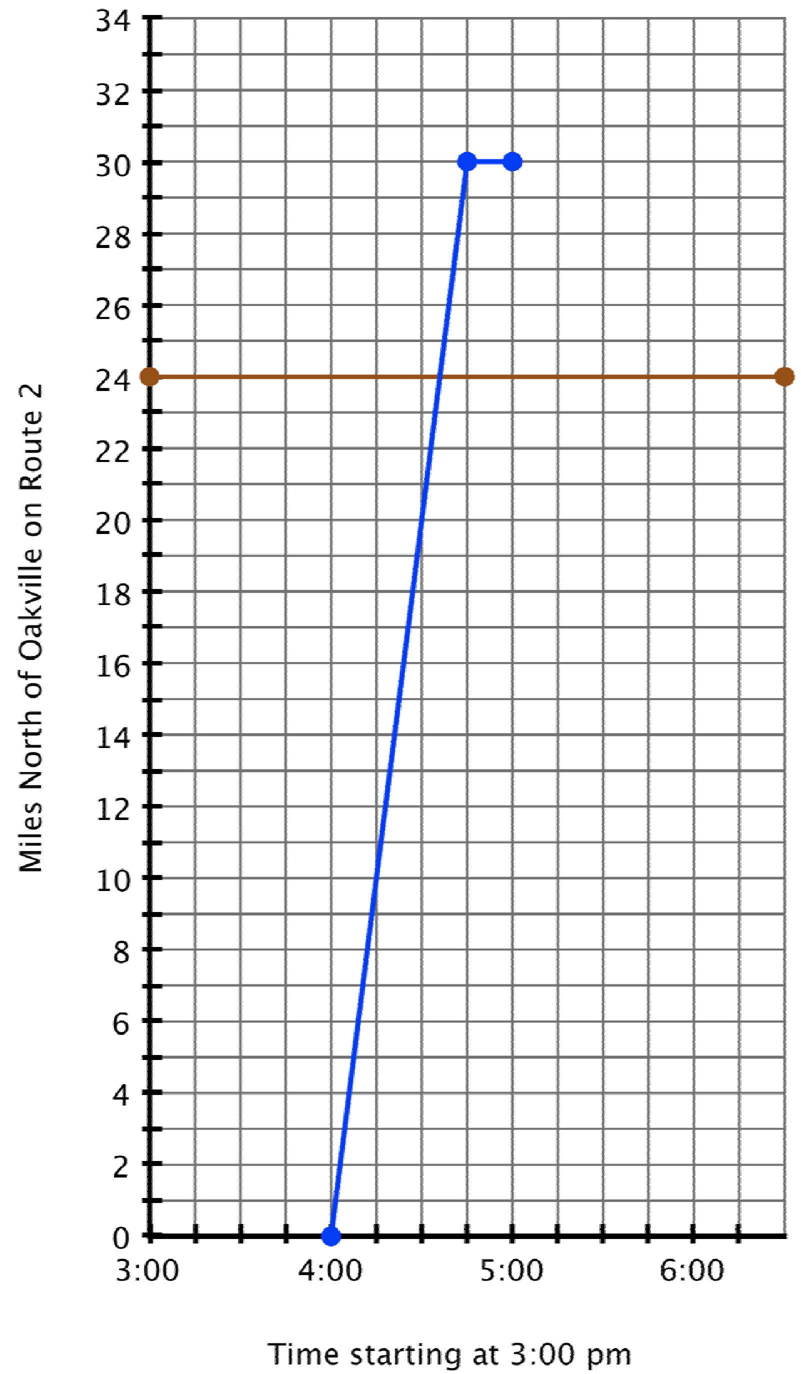
TOM	DORA
Tom lives 17 miles north of Oakville. He left home at 4:00 p.m. and walked 9 miles north to arrive at Curley's Burger Stand for dinner at 6:15 p.m. Draw Tom's graph in GREEN.	Dora lives 12 miles north of Oakville. At 3:00 p.m., she left home and rode her bicycle north at 9 miles per hour to the bus stop in Albany. Draw Dora's graph in RED.
HARRY	**JOAN**
Harry left his house, 2 miles north of Oakville, at 3:30 p.m. He drove north for a half an hour at 36 miles per hour. He stopped at the Science Museum for 45 minutes. He then drove to Curley's Burger Stand, 26 miles from Oakville, arriving there at 5:00. Draw Harry's graph in ORANGE.	Joan was at her house, across the street from the Science Museum, 20 miles north of Oakville until 5:00 p.m. She then jogged north for one hour at 6 miles per hour. She stopped At Curley's Burger Stand for a soda and met a friend. Draw Joan's graph in PURPLE.

Who Stole the Pie? Graph

On the graph, plot the travels of the bus, Tom, Dora, Harry, and Joan. **Assume everyone traveled constant rates.**



Getting Started Questions

Answer these questions about the graph as you begin to solve the mystery. Use extra paper if needed to give complete answers to each of the questions.

1. Where are Oakville and Albany on the graph?

2. What is the meaning of the horizontal line at the point 24 miles north?

3. How is the x-axis different from other x-axes you've worked with?

4. Where would you find 4:30 on the x-axis?

5. Why does the line for the bus have a horizontal section?

6. How would you graph the bus's return trip?

Question Set I

Use your graph to answer these questions and solve the mystery. Use extra paper if needed to give complete answers to each of the questions.

1. Which of the suspect's graphs did you find easiest to graph? Explain why.

2. Which of the suspect's graphs did you find most challenging to graph? Explain why.

3. Who stole the apple pie? Write a convincing explanation about how you know who is guilty and who is innocent.

Question Set II

Use your graph from *Who Stole the Apple Pie* to answer these questions. You need to be able to explain your answers.

1. Where are each of the characters in this problem (Tom, Dora, Harry, Joan, the bus, and the apple pie) at 4:15? Explain how you know this.

2. Who stops at Curley's Burger Stand? How does the graph help you see when each person arrived at Curley's Burger Stand?

3. When did Dora bike past Curley's burger stand? How can you figure this out from the graph?

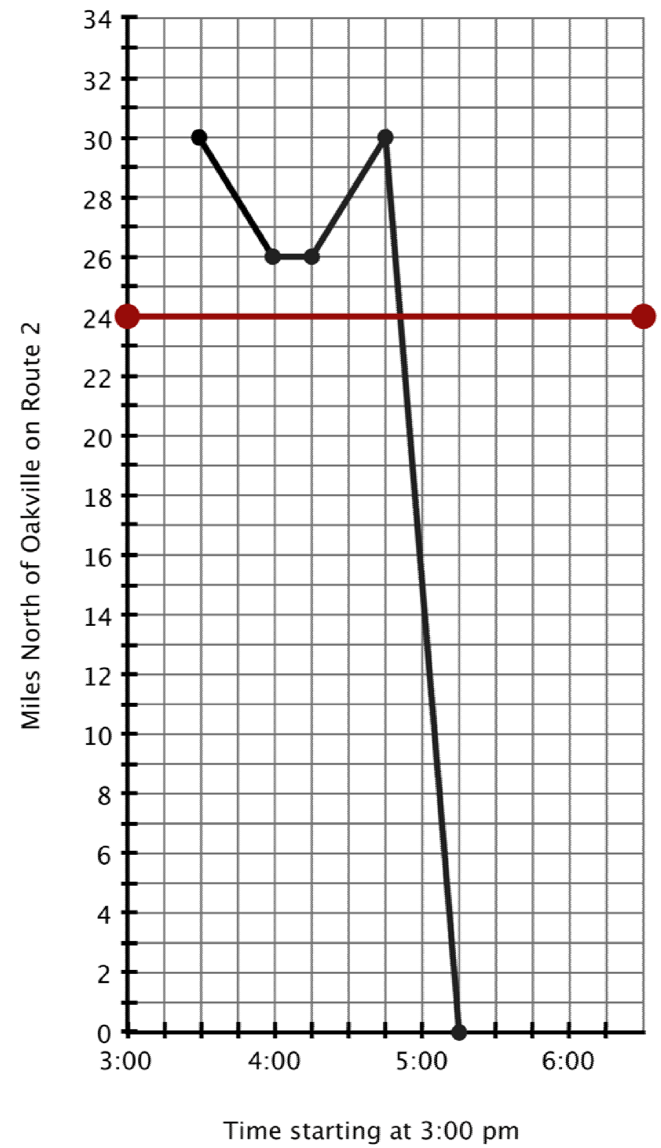
4. Does Harry drive at the same speed from his house to the museum as he drives from the museum to Curley's? Explain how you figured this out.

5. Who does Tom see on his walk? When does Tom see the bus pass him?

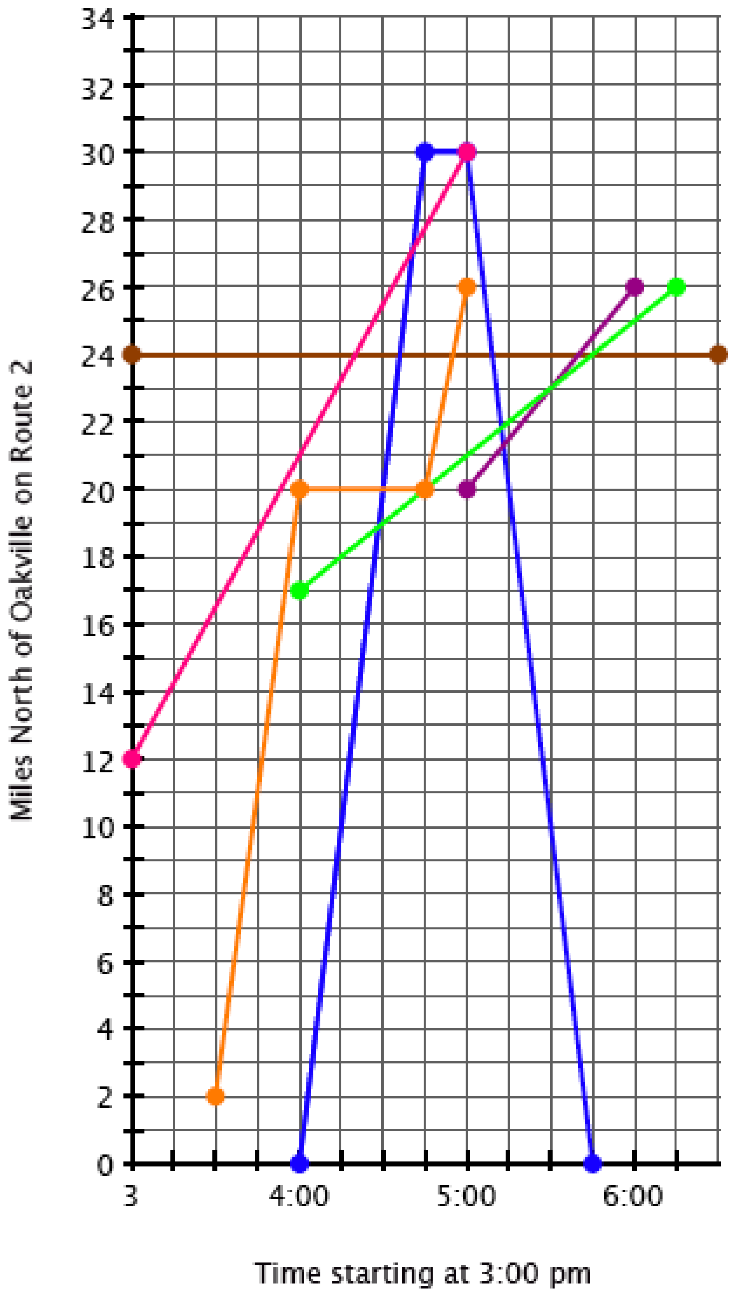
Question Set III

1. Write two questions (similar to those in Question Set II) that can be answered using this graph. Include the answers after your questions.

2. You find out that Edward is also a suspect and here is his “graph” (on the same axes but without all of the other graphs shown). Write a story to fit his graph and decide if he could have been the thief.



Solution Graph



Those are the lessons.

But, you also want teachers to develop the right orientation to teaching them.

One Sample Technique:

Teachers Interview Students

The teacher picks a kid who's in trouble mathematically.

"I picked this student to interview because I didn't think she belonged in the class. All she every produced were meaningless chicken scratches on the page. I wanted to see just what she knew."

The teacher gives the (second language) student a word problem.

"A five-pound box of sugar costs $1.80 and contains 12 cups of sugar. Marella and Mark are making a batch of cookies. The recipe calls for 2 cups of sugar. Determine how much the sugar for the cookies costs."

What the teacher said:

"The student read the problem and figured out what to do, perfectly. She has the concepts! To finish the problem she had to do a division. She tried, and produced meaningless chicken scratches. You know what, she belongs in my class - she just needs remediation on the procedures!"

And then…

"I had a completely wrong impression of her.

Oh my god, I'm going to have to interview all my students!"

But, more broadly, how do we plan to introduce our formative lessons to an entire district?

(This is the challenge of the SERP work with San Francisco.)

We work with teacher partners, known as co-developers.

A teacher co-developer teaches the lesson.

We tape:

- a pre-interview,
- the lesson, focusing on kids, including copies of student work.
- a post-interview

We create…

- Packages of materials containing the problems, annotated student work, and teacher comments.
- Video records (from the taped lessons) that serve as opportunities for focusing on student thinking and how to build on it.

Other teacher co-developers at the same grade level…

Teach the same lessons and annotate the resources with their own comments…

ultimately producing shareable resources grounded in instruction.

If this all works the way we hope, what it produces should be usable by other teachers in the district.

At least we hope so - time will tell.

But what about broader dissemination?

That’s the Gates grant. We’re producing 20 formative assessment lessons per grade, targeted to central concepts.

The idea is to provide enough support so that teachers develop habits of mind consistent with diagnostic teaching, and the resources to do it effectively.

That’s what we hope - as above, time will tell.

I welcome comments and questions.